Combinations

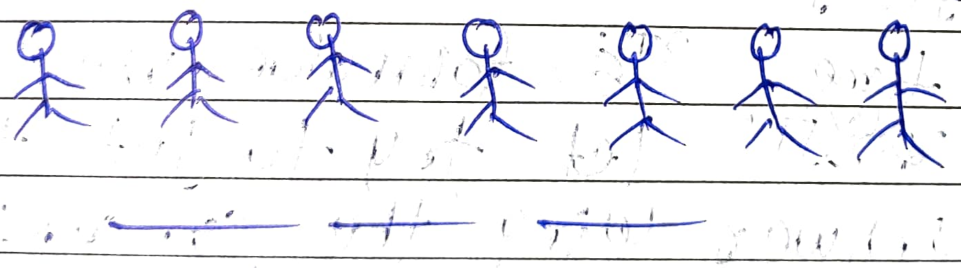
⇒ combinations का मतलब है "selections".

for example :-

मान लिजिए मेरे पास 7 students हैं, in how many ways can i select 3 students out of 7 given students.

तो इसको logically derive करने के लिए मैं permutations का use करता हूँ i know how to calculate permutation.

So permutations को use कर के मैं combinations निकालता हूँ।



So first देखिए, मान लिजिए मेरे पास 7 students हैं; assume that i have three places.

The question is in how many ways 3 students out of these 7 students can

be arranged in these three places मान लिजिए यह question है जो की originally permutation का question है।

तो यह permutation को हम दो तरीको से निकालेंगे :-

(i) using the formulae

No. of arrangements of 3 students out of 7 students

$= {}^7P_3$

* अब मैं क्या करूंगा second time यह formulae use नहीं करूंगा let try to get the answer using the concept of Multiplication rule.

(ii) Multiplication rule

मान लिजिए 7 students मे 3 student को यहाँ खड़ा करना है तो मै क्या करूँगा इस बार Multiplication theorem use करता हूँ।

पहले case मे जो $^7p_3$ आया वो भी Multiplication theorem से हि आया कैसे 1st place मे 7 लोगो को रख सकता हूँ; (मे से कोई एक) next place मे 6 लोगो मे से कोई एक को रख सकता हूँ & last मे 5 लोगो मे से कोई एक को रख सकता हूँ इसलिए effectively answer आ गया $7 \times 6 \times 5$.

Now i use the same multiplication theorem in a different way.

मुझे क्या करना है, तीन places मे arrange करना है लोगो को, तो i have 7 students तो instead of calling one by one से $^7p_3$ कैसे आया, first place मे 7 मे से कोई एक को बुलाया place कर लिया; उसके बाद second place

के लिए six options बनाया place किया उसके बाद third place के लिए five options

This time what i will do i will not call each of them seperately मैं क्या करता हूँ आप में से कोई भी तीन लोग बाहर आ जाएँ अब अलग-अलग तरीके से बाहर आ सकते हैं; जितना तरीका होता है वही number of selection होता है

मान लिजिए अब मैं पूछता हूँ आप से 3 of you come out; in how many ways 3 of you can come out अभी मैं नहीं जानता let it be 'x'.

No. of selections of 3 students out of 7 students = 'x'

so now मान लिजिए तीन लोग बाहर आ जाए अब मैं क्या करता हूँ- की तीन लोगो को 3 places मे कितना तरीके से arrange कर सकता हूँ 3! ways so, first selection के 3! arrangements आएगा ; second selection के लिए 3! arrang.s आएगा & third selection के लिए 3! arrangements आएगा वैसे ही x selection के लिए $x \cdot 3!$

i.e. No. of arrangements

$$= x \cdot 3!$$

तो first Method से answer $^7P_3$ आया & second method से answer $x \cdot 3!$ आया ; जहाँ 'x' क्या है no. of selections of 3 students. तो ये दोनो तो same होना चाहिए इसलिए इन दोनो को equate कर देंगे।

$$^7P_3 = x \times 3!$$

$$\Rightarrow x = \frac{^7P_3}{3!} = {}^7C_3$$

Sol:

No. of selections is same as No. of permutation divided by r!

i.e. $^{n}C_{r} = \frac{^{n}P_{r}}{r!}$

So you see the relation b/w permutation & combination

→ no. of arrangements divided by r! तो no. of selections आएगा।

→ no. of selections multiplied by r! करने से no. of arrangements आएगा।

* No. of selections or combinations of 'r' objects out of 'n' different objects

$$= \frac{^{n}P_{r}}{r!} = {}^{n}C_{r}$$

Some points on ${}^nC_r$:-

⟨1.⟩ $${}^nC_r = \frac{n!}{(r!) \times (n-r)!}$$

⟨2.⟩ $${}^nC_r = \frac{n(n-1)(n-2) \cdots \text{ to } r \text{ factors}}{r!}$$

⟨3.⟩ $${}^nC_n = {}^nC_0 = 1$$

⟨4.⟩ $${}^nC_r = \frac{{}^nP_r}{r!}$$

Properties of ${}^nC_r$:-

⟨1.⟩ $${}^nC_r = {}^nC_{n-r}$$ for $[0 \leq r \leq n]$

⟨2.⟩ $${}^nC_x = {}^nC_y$$

only when
(i) $x = y$
(ii) $x + y = n$

⟨3.⟩ $${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$ for $[1 \leq r \leq n]$

4. $\dfrac{{}^{n}C_{r-1}}{{}^{n-1}C_{r-1}} = \dfrac{n}{(n-r+1)}$

5. $\dfrac{{}^{n}C_{r}}{{}^{n-1}C_{r-1}} = \dfrac{n}{r}$

6. $\dfrac{{}^{n}C_{r}}{{}^{n}C_{r+1}} = \dfrac{r+1}{n-r}$

Q. If ${}^{n}C_8 = {}^{n}C_6$, find ${}^{n}C_3$.

Ans: हमे पता है कि ${}^{n}C_p = {}^{n}C_q$ &

$p \neq q \Rightarrow n = p+q$.

$\therefore {}^{n}C_8 = {}^{n}C_6$

$\Rightarrow n = 8+6 = 14$.

Hence

$${}^{14}C_3 = \frac{14 \times 13 \times 12}{3!} = \frac{14 \times 13 \times \cancel{12}^{2}}{\cancel{3 \times 2 \times 1}}$$

$$= 26 \times 14$$
$$= 364$$

Q. If ${}^{n}C_{r-1} : {}^{n}C_r : {}^{n}C_{r+1} = 3:4:5$, find the values of n and r.

Ans: अब मेरे पास है

$$\frac{{}^{n}C_r}{{}^{n}C_{r+1}} = \frac{r+1}{n-r} \quad \text{------ (i)}$$

अब question से यह information है मेरे पास :-

$$\frac{{}^{m}C_{r}}{{}^{m}C_{r+1}} = \frac{4}{5}$$

↳ इसको हम (i) मे put किए तो हमको मिला :-

$$\frac{r+1}{m-r} = \frac{4}{5}$$

$\Rightarrow 5r + r = 4m - 4r$

$\Rightarrow 4m - 9r = 5$ ——— (ii)

'r' को (r−1) से replace किए (i) मे तो मुझे मिला,

$$\frac{{}^{m}C_{r-1}}{{}^{m}C_{r}} = \frac{r}{m-r+1} \quad \text{——— (iii)}$$

अब question से यह information है मेरे पास :-

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$$\frac{^{m}C_{r-1}}{^{m}C_{r}} = \frac{3}{4}$$

↳ इसको हम (iii) में put किए तो हमको मिला :-

$$\frac{r}{m-r+1} = \frac{3}{4}$$

$\Rightarrow 3m - 3r + 3 = 4r$ —— (iv)

$\Rightarrow 3m - 7r = -3$

On solving (ii) & (iv), we get

$m = 62$ & $r = 27$.

Q. If ${}^{2m}C_3 : {}^{m}C_3 = 11:1$; find 'm'.

Ans:

$${}^{2m}C_3 = \frac{2m(2m-1)(2m-2)}{3!}$$

$$= \frac{2m(m-1)(2m-1)}{3}$$

&

$${}^{m}C_3 = \frac{m(m-1)(m-2)}{3!} = \frac{m(m-1)(m-2)}{6}$$

Hence

$$\frac{{}^{2m}C_3}{{}^{m}C_3} = \frac{2m(m-1)(2m-1)}{3} \times \frac{6}{m(m-1)(m-2)}$$

$$= \frac{4(2m-1)}{(m-2)}$$

अब

$$\frac{{}^{2m}C_3}{{}^{m}C_3} = \frac{11}{1} \Rightarrow \frac{4(2m-1)}{(m-2)} = \frac{11}{1}$$

$\Rightarrow 8n - 4 = 11n - 22$

$\Rightarrow 3n = 18$

$\Rightarrow n = \frac{18}{3} 6$ $\therefore \boxed{n = 6}$

Q. If ${}^{n}P_{r} = {}^{n}P_{r+1}$ & ${}^{n}C_{r} = {}^{n}C_{r-1}$, find n and r.

Ans: ${}^{n}P_{r} = \frac{n!}{(n-r)!}$ —— (i)

& ${}^{n}P_{r+1} = \frac{n!}{(n-r-1)!}$ —— (ii)

~~on~~ since the relation is

$${}^{n}P_{r} = {}^{n}P_{r+1}$$

$\Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow (n-r)! = (n-r-1)!$

$\Rightarrow (n-r) \times (\cancel{n-r}-1)! = (\cancel{n-r}-1)!$

$\Rightarrow n - r = 1$ —— (i)

$\because {}^nC_r = {}^nC_{r-1} \Rightarrow \dfrac{{}^nC_r}{{}^nC_{r-1}} = 1$

$\Rightarrow \dfrac{n-r+1}{r} = 1$

$\Rightarrow n - 2r = -1$ —— (ii)

On solving (i) & (ii), we get

$n = 3$ and $r = 2$.

Q. Prove that :-

$${}^{2n}C_n = \frac{2^n \times [1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)]}{n!}$$

Ans: L.H.S. $= {}^{2n}C_n$

$$= \frac{(2n)!}{(n!)\cdot(2n-n)!} = \frac{(2n)!}{(n!)^2}$$

$$= \frac{[(2n)(2n-1)(2n-2)\cdots 4\cdot 3\cdot 2\cdot 1]}{(n!)^2}$$

$$= \frac{[(2n)(2n-2)(2n-4)\cdots 4\cdot 2]\times[(2n-1)(2n-3)\cdots 5\cdot 3\cdot 1]}{(n!)^2}$$

$$= \frac{2^n[n(n-1)(n-2)\cdots 2\cdot 1]\times[(2n-1)(2n-3)\cdots 5\cdot 3\cdot 1]}{(n!)^2}$$

$$= \frac{2^n\times(n!)\times[1\cdot 3\cdot 5\cdots(2n-3)(2n-1)]}{(n!)^2}$$

$$= \frac{2^n\times[1\cdot 3\cdot 5\cdots(2n-3)(2n-1)]}{(n!)}$$

hence proved

Q Prove that :-

$$^{4n}C_{2n} : {}^{2n}C_n = [1\times3\times5\times\cdots\times(4n-1)] : [1\times3\times5\times\cdots\times(2n-1)]^2$$

Ans: मेरे पास दिया है :-

$$^{4n}C_{2n} = \frac{(4n)!}{(2n)!\times(2n)!} \times \frac{(n!)\times(n!)}{(2n)!}$$

$$= \frac{(4n)!\times(n!)^2}{(2n)!\times[(2n!)]^2}$$

$$= \frac{[1\cdot2\cdot3\cdots(4n-1)(4n)]\times(n!)^2}{(2n)!\times[1\cdot2\cdot3\cdot4\cdots(2n-2)(2n-1)(2n)]^2}$$

$$= \frac{[1\cdot3\cdot5\cdots(4n-1)]\times[2\cdot4\cdot6\cdots(4n)]\times(n!)^2}{(2n)!\,[1\cdot3\cdot5\cdots(2n-1)]^2\times2^{2n}\times(n!)^2}$$

$$= \frac{[1\cdot3\cdot5\cdots(4n-1)]}{[1\cdot3\cdot5\cdots(2n-1)]^2}$$

(P.T.O.)

$\therefore {}^{4n}C_{2n} : {}^{2n}C_{n} = [1\cdot 3\cdot 5 \cdots (4n-1)] :$
$[1\cdot 3\cdot 5 \cdots (2n-1)]^2$

hence proved

Q. If ${}^{m+2}C_8 : {}^{m-2}P_4 = 57:16$, find m ?

Ans: मेरे पास दिया है कि:-

$${}^{m+2}C_8 = \frac{(m+2)!}{(8!) \times (m+2-8)!} = \frac{(m+2)!}{(8!)(m-6)!}$$

$$\left\{ \because {}^{m}C_{r} = \frac{m!}{(r!) \times (m-r)!} \right\}$$

(2)

$${}^{m-2}P_4 = \frac{(m-2)!}{(m-2-4)!} = \frac{(m-2)!}{(m-6)!}$$

$$\left\{ \because {}^{m}P_{r} = \frac{m!}{(m-r)!} \right\}$$

Now,

$$\frac{{}^{m+2}C_8}{{}^{m-2}P_4} = \frac{57}{16}$$

$$\Rightarrow \frac{(n+2)!}{(8!)\times(n-6)!} \times \frac{(n-6)!}{(n-2)!} = \frac{57}{16}$$

$$\Rightarrow \frac{(n+2)(n+1)\,n(n-1)(n-2)!}{(n-2)!\times 8!} = \frac{57}{16}$$

$$\Rightarrow (n+2)(n+1)\,n\,(n-1) = \frac{57}{16}\times 8!$$

$$\Rightarrow (n+2)\cdot(n+1)\,n\,(n-1) = \frac{57}{16}\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1$$

$$= (n+2)(n+1)(n)(n-1) = 21\times 20\times 19\times 18$$

$$\Rightarrow n-1 = 18 \quad \Rightarrow \boxed{n=19}$$

Hence

$$n = 19$$

Q. If ${}^{n}C_7 = {}^{n}C_5$, find n.

Ans: $\{\because {}^{n}C_x = {}^{n}C_y$ only when $n = x+y;\ x \neq y$

$\Rightarrow n = 7+5$
$= 12$ $\quad \therefore n = 12$

Q. If ${}^{n}C_{14} = {}^{n}C_{16}$, find n.

Ans: $\{\because {}^{n}C_x = {}^{n}C_y$ only when $n = x+y;\ x \neq y\}$

$\Rightarrow n = 14+16$
$= 30$ $\quad \therefore n = 30$

Q. If ${}^{n}C_{16} = {}^{n}C_{19}$, find n.

Ans: $\{\because {}^{n}C_x = {}^{n}C_y$ only when $n = x+y;\ x \neq y\}$

$\Rightarrow n = 19+16$
$= 45$

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Practical problems on Combinations

Q. In how many ways can a cricket eleven be chosen out of a batch of 15 players, if:

(i.) there is no restriction on the selection?

(ii.) a particular player is always chosen?

(iii.) a particular player is never chosen?

Ans: (i.) Total no. of ways of selecting 11 players out of 15.

$\Rightarrow {}^{15}C_{11} = {}^{15}C_{15-11} = {}^{15}C_4$

$= \frac{15 \times 14 \times 13 \times 12}{\lfloor 4}$

$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365.$

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(ii.) जब एक player हमेसा choose करा जाना है, तो उसका मतलब यह है की हमें 10 player अब चुनने हैं बचे 14 मे से (क्योंकी एक तो already selected है।)

$\therefore$ required number of ways

$$= {}^{14}C_{10} = {}^{14}C_{14-10} = {}^{14}C_4$$

$$= \frac{14 \times 13 \times 12 \times 11}{4 \times 3 \times 2 \times 1} = 1001$$

(iii.) जब एक player हमेसा नही चुना जाता है, अब हमे 11 player चुनना है बचे 14 players मे से।

$\therefore$ required number of ways

$$= {}^{14}C_{11} = {}^{14}C_{14-11} = {}^{14}C_3$$

$$= \frac{14 \times 13 \times 12}{3 \times 2 \times 1} = 364$$

Q. In how many ways can a committee of 5 members be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies?

Ans: हमें select करना है
(3 men 6 Men's में से) and
(2 ladies 5 ladies में से)

∴ required no. of ways

$= ({}^6C_3 \times {}^5C_2)$

$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{5 \times 4}{2 \times 1}$

$= (20 \times 10) = 200$

Q. Out of 5 men and 2 women, a committee of 3 is to be formed. In how many ways can it be formed if at least one woman is to be included?

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Ans: Commitee बनता है by choosing
(i.) 1 woman and 2 men; (or)
(ii.) 2 women and 1 man.

अत

No. of ways of choosing (1 woman out of 2) and (2 men out of 5)

$$=({}^2C_1 \times {}^5C_2) = \frac{2 \times 5 \times 4}{2 \times 1} = 20$$

(or),

No. of ways of choosing (2 women out of 2) and (1 man out of 5)

$$=({}^2C_2 \times {}^5C_1) = 1 \times 5 = 5$$

Hence,

Total number of ways of forming the committee

$$=(20 + 5) = 25.$$

(b) at most 2 ladies are included

यह दो तरीके से हो सकता है

(1 lady out of 4) & (4 men out of 6)

or (2 ladies out of 4) & (3 men out of 6)

∴ The no. of ways of these selections are :—

$= ({}^4C_1 \times {}^6C_4) + ({}^4C_2 \times {}^6C_3)$

$= (4 \times 15) + (6 \times 20)$

$= 60 + 120$

$= 180$

Q An examination paper containing 12 questions consists of two parts, A and B. Part A contains 7 questions and Part B contains 5 questions. A candidate is required to attempt 8 questions, selecting at least 3 from each part. In how many ways can the candidate select the questions?

Ans: स्पष्ट है कि candidate इन mentioned ways से questions को select कर सकता है।

(i) (3 out of 7 from A) and (5 out of 5 from B)

or (ii) (4 out of 7 from A) and (4 out of 5 from B)

(iii) (5 out of 7 from A) and (3 out of 5 from B)

∴ Number of ways of these selections are:

$$= \left({}^7C_3 \times {}^5C_5\right) + \left({}^7C_4 \times {}^5C_4\right) + \left({}^7C_5 \times {}^5C_3\right)$$

$$= 35 + 175 + 210$$

$$= 420.$$

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Q. For the post of 5 teachers, there are 23 applicants. 2 posts are reserved for SC candidates and there are 7 SC candidates among the applicants. In how many ways can the selection be made?

Sol: माना है कि मेरे पास, 7 SC candidates & 16 non-SC candidates.

इसलिए हमे selection करना है (2 out of 7) and (3 out of 16).

Hence,

No. of ways of making the selection

$= {}^{7}C_{2} \times {}^{16}C_{3}$

$= \frac{7 \times 6}{2} \times \frac{16 \times 15 \times 14}{3 \times 2 \times 1}$

$= 11760$

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Q. A box contains 6 red and 5 white balls. In how many wa[ys] can 6 balls be selected so tha[t] there are at least two balls of each colour?

Ans: साफ है कि हम 6 ball को following ways से select कर सकेंगे जैसे कि कम से कम दो ball हर एक colour का हो :-

(i) (2 red balls out of 6) & (6 white balls out of
or (ii) (3 red balls out of 6) & (3 white balls out o[f]
or (iii) (4 red balls out of 6) & (2 white balls out of

∴ No. of ways of these selections are.

$$= ({}^6C_2 \times {}^5C_4) + ({}^6C_3 \times {}^5C_3) + ({}^6C_4 \times {}^5C_2)$$

$$= 75 + 200 + 150$$
$$= 425.$$

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Q. How many committes of 5 persons with a chairperson can be selected from 12 persons?

Ans: देखो इस question में यह confusion हो सकती है कि 5 person + 1 chairman या 4 person + 1 chairman तो इसका जवाब है की बाद वाला सही है यानी 4 person को select करना है और एक chairperson को select करना है।

No. of ways of selecting a chairperson out of 12 persons

$= {}^{12}C_1 = 12$

No. of ways of selecting 4 persons out of remaining 11 persons

$= {}^{11}C_4 = \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1}$

$= 330$

Hence,

Required no. of ways

$= (12 \times 330)$

$= 3960$

Q. A box contains 3 black, 2 white and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

Ans: तो question के मांग के अनुसार हम data लिखते हैं :—

We have

3 black and 6 non-black balls

हम select कर सकते हैं

(i) (1 black balls out of 3) & (2 non-black balls out of 6)

(or) (ii) (2 black balls out of 3) & (1 non-black balls out of 6)

(or) (iii) (3 black balls out of 3).

$= ({}^3C_1 \times {}^6C_2) + ({}^3C_2 \times {}^6C_1) + ({}^3C_3)$

$= 45 + 18 + 6 = 69.$

Q. A group consists of 7 boys and 4 girls. In how many ways can a team of 5 members be selected, if the team has:

(i.) no girls?

(ii.) at least 1 boy and 1 girl?

(iii.) at least 3 girls?

Ans: (i) if no girls to be selected

तो अब हमें 5 boys select करने है 7 boys में से :-

∴ required no. of ways

$= {}^7C_5 = {}^7C_{7-5} = {}^7C_2$

$= \frac{7 \times 6^3}{2 \times 1} = 21$

(ii) when at least 1 boy and 1 girl

यह case below mentioned तरीके से हो सकता है :-

(1 boy and 4 girls) or (2 boys and 3 girls) or (3 boys and 2 girls) or (4 boys and 1 girls)

∴ required No. of ways.

$$= ({}^7C_1 \times {}^4C_4) + ({}^7C_2 \times {}^4C_3) + ({}^7C_3 \times {}^4C_2) + ({}^7C_4 \times {}^4C_1)$$

$$= 7 + 84 + 210 + 140$$

$$= 441.$$

(P.T.O.)

(iii) when at least 3 girls are taken

यह हम इन तरीको से कर सकते हैं:-

(3 girls and 2 boys) or (4 girls and 1 boy)

∴ required no. of ways

$= ({}^4C_3 \times {}^7C_2) + ({}^4C_4 \times {}^7C_1)$

$= ({}^4C_1 \times {}^7C_2) + (1 \times {}^7C_1)$

$= 4 \times \frac{7 \times 6}{2 \times 1} + (1 \times 7)$

$= 12 \times 7 + 7$

$= 84 + 7$

$= 91.$

Q. A bag contains 6 white marbles and 5 red marbles. Find the no. of ways in which 4 marbles can be drawn from the bag, if

(a.) they can be of any colour;
(b.) 2 must be white and 2 red
(c.) they must all be of the same colour.

Ans:

(a.) Drawing 4 marbles of any colour:

Total no. of marbles = (6+5)
= 11

इस case में हमें 4 marbles 11 marbles में से निकालना है।

∴ required no. of ways

$= {}^{11}C_4$

$= \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1}$

= 330.

(b.) Drawing 2 white and 2 red marbles
यह हम ऐसे draw कर सकेंगे

(2 white marbles out of 6) and
(2 red marbles out of ~~6~~ 5)

∴ required number of ways

$= \left({}^6C_2 \times {}^5C_2\right)$

$= \frac{6 \times 5}{2 \times 1} \times \frac{5 \times 4}{2 \times 1}$

$= 25 \times 6 = 150$

(c.) Drawing 4 marbles of the same colour
हम ऐसे draw कर सकेंगे

(4 white marbles out of 6) or (4 red marbles out of 5)

(P.T.O.)

∴ required number of ways

$$= ({}^6C_4 + {}^5C_4)$$

$$= {}^6C_2 + {}^5C_1$$

$$= \left(\frac{6 \times 5}{2 \times 1} + 5\right)$$

$$= 20$$

Q. We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can the selection be made?

Ans: हमारे पास दो cases है:
या तो (either A is chosen)
(or) (A is not chosen)

(P. T. O.)

Case I: जब A choo…
इस case में A …
B भी choose …

अब, हमें 4 person…
है बचे …

∴ required no. of …

$$= {}^6C_4 = {}^6C…$$

Case II: जब A …

इस case में …
choose करना …

∴ required …

Combining both …

Total no. of …

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Case I: जब A choose कर लिया गया

इस case में A भी choose हुआ और B भी choose हुआ.

अब, हमें 4 persons और choose करने है बचे हुए 6 में से।

∴ required no. of ways

$$= {}^6C_4 = {}^6C_2 = \frac{\cancel{6}^3 \times 5}{\cancel{2} \times 1} = 15.$$

Case II: जब A नहीं choose करेंगे

इस case में हमें 6 persons को choose करना है 7 persons में से।

∴ required no. of ways $= {}^7C_6 = {}^7C_1 = 7$

Combining both the cases, we have

Total no. of ways $= (15+7) = 22.$

Q. Find the number of ways of choosing 4 cards from a pack of 52 playing cards. In how many of these

(i) 4 cards are of the same suit?
(ii) 4 cards belong to four different suits?
(iii) 4 cards are face cards?
(iv) 2 are red cards and 2 are black cards?
(v) cards are of the same colour?

Ans:-

Diamond	Spade	Club	Heart
13 cards	13 cards	13 cards	13 cards

Face cards = 12

Red cards = 26

Black cards = 26

Number of ways of choosing 4 cards out of 52.

$$= {}^{52}C_4$$

$$= \frac{52 \times 51 \times 50 \times 49}{4 \times 3 \times 2 \times 1}$$

$$= 270725$$

(i) Choosing 4 cards of the same suit :

हमें draw करना है ऐसे :-

(4 cards from 13 cards of Diamond)
or (4 cards from 13 cards of spade)
or (4 cards from 13 cards of club)
or (4 cards from 13 cards of heart).

∴ required no. of ways.

$$= \left({}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 \right)$$

$$= 4 \times {}^{13}C_4$$

$$= \frac{4 \times 13 \times 12 \times 11 \times 10}{4 \times 3 \times 2 \times 1}$$

$$= 2860$$

(ii) Choosing 4 cards from 4 different suits

हमें draw करना है ऐसे :-

(P.T.O.)

(1 card from 13 cards of diamond)
and (1 card from 13 cards of spade)
and (1 card from 13 cards of club)
and (1 card from 13 cards of heart)

∴ required no. of ways

$$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$$

$$= 13 \times 13 \times 13 \times 13$$

$$= (13)^4 \text{ ways}$$

(iii.) Choosing 4 cards out of 12 face cards

हमें 4 cards चुनने हैं 12 face cards में से

∴ required no. of ways.

$$= {}^{12}C_4 = \frac{\overset{2}{\cancel{12}} \times 11 \times \overset{5}{\cancel{10}} \times 9}{\underset{2}{\cancel{4}} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 45 \times 11 = 495$$

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(iv.) choosing 2 red cards & 2 black cards

हमें 2 card चुनने हैं हर एक pack of 26 red cards & 26 black cards से

∴ reqd. no. of ways

$= \left({}^{26}C_2 \times {}^{26}C_2\right)$

$= \left(\frac{26 \times 25}{2}\right)^2 = 325 \times 325$

$= 105625$

(v.) choosing all the 4 cards of the same colour

हमें 4 cards choose करने हैं 26 black cards में से या 4 cards choose करने हैं 26 red cards में से

∴ required no. of ways

$= {}^{26}C_4 + {}^{26}C_4$

$= 2 \times {}^{26}C_4 = 2 \times \frac{26 \times 25 \times 24 \times 23}{4 \times 3 \times 2 \times 1}$

$= 29900$

Q. From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can they be chosen?

Ans: एक class में 25 बच्चे हैं। 10 को choose करना है party पर जाने के लिए

3 बच्चों ने यह decide किया कि या तो सभी जाएंगे या कोई नहीं जाएगा।

Case I जब 3 particular students join the party:

इस case में हमें 7 बच्चे choose करने हैं 22 बच्चों में से।

यह होगा in $^{22}C_7$ ways.

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Case II: जब 3 particular students party नहीं join करें।

इस case में, हमें 10 students choose करने हैं 22 students में से

यह होगा $^{22}C_{10}$ ways में।

Hence, the required No. of ways

$$=\left({}^{22}C_7 + {}^{22}C_{10}\right) = 817190$$

Q. A boy has 3 library ticket and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics Part II, unless Mathematics Part I is also borrowed. In how many ways can he choose the 3 books to be borrowed?

माना है कि boy या तो Mathematics Part-II borrow कर सकता है या नहीं कर सकता।

Case I: जब एक boy Mathematics Part-II borrow करता है।

अब, इस case में वह Mathematics Part-I भी borrow करेगा।

क्योंकि उसके पास 3 library tickets है,

इसलिए वह 1 book borrow कर सकता है बचे हुए 6 books में से।

यह काम $^6C_1 = 6$ ways में होगा।

Case II: जब एक boy Mathematics Part-II नहीं borrow करेगा

तब, वह 3 books borrow करेगा बचे हुए 7 books में से।

यह काम $^7C_3$ ways मे होगा

$$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35 \text{ ways.}$$

Hence,

The required no. of ways $= (6+35)$
$= 41$ ways.

In an examination, a candidate has to pass in each of the 5 subjects. In how many ways can he fail?

Candidate जो है वह fail कर सकता है by failing in :-

(i) 1 subject
(ii) 2 subjects
(iii) 3 subjects
(iv) 4 subjects
(v) 5 subjects

∴ Total no. of ways in which he can fail

$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= {}^5C_1 + {}^5C_2 + {}^5C_2 + {}^5C_1 + {}^5C_5$

$= (5 + 10 + 10 + 5 + 1)$ $\left\{ \because {}^nC_r = {}^nC_{n-r} \right\}$

$= 31$.

Q. Determine the no. of 5-card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Ans: ∵ There are 1 ace card in each of the deck.

No. of ways of selecting 1 ace out of 4 aces

$= {}^{4}C_{1} = 4$

No. of ways of selecting 4 cards out of remaining 48.

$= {}^{48}C_{4} = \frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2 \times 1}$

$= 194580$.

Hence,

Required no. of combinations

$= 194580 \times 4$

$= 778320$

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Q. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl, if each cricket team of 11 must include exactly 4 bowlers?

Ans: No. of ways of selecting 4 bowlers out of 5.

$$= {}^5C_4 = {}^5C_{(5-4)} = {}^5C_1 = 5.$$

No. of ways of selecting 7 batsmen out of 12.

$$= {}^{12}C_7 = {}^{12}C_{12-7} = {}^{12}C_5$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= 66 \times 12$$
$$= 792$$

Hence,
The no. of ways of selecting the team
$= (5 \times 792) = 3960$ ways.

Q. A committee of 12 persons is to be formed from 9 women and 8 men. In how many ways can this be done if at least 5 women have to be included in a commitee? In how many ~~ways~~ of these committes are

(a.) the women in majority?

(b.) the men in majority?

Ans: देखो 9 women & 8 men है अगर मुझे एक comittee form करना है जिसमे कम से कम 5 women होनी चाहिए, हम इसे ऐसे choose करेंगे :-

(i.) 5 women and 7 men (or)

(ii.) 6 women and 6 men (or)

(iii.) 7 women and 5 men (or)

(iv.) 8 women and 4 men (or)

(v.) 9 women and 3 men

(P.T.O.)

Hence,

The total no. of ways of forming the committee

$= ({}^9C_5 \times {}^8C_7) + ({}^9C_6 \times {}^8C_6) + ({}^9C_7 \times {}^8C_5) + ({}^9C_8 \times {}^8C_4) + ({}^9C_9 \times {}^8C_3)$

$= 1008 + 2352 + 2016 + 630 + 56$

$= 6062.$

(a) साफ है कि women majority में है (iii), (iv) & (v) cases में

इसलिए, the number of committees in which the women are in majority

$= ({}^9C_7 \times {}^8C_5) + ({}^9C_8 \times {}^8C_4) + ({}^9C_9 \times {}^8C_3)$

$= ({}^9C_2 \times {}^8C_3) + ({}^9C_1 \times {}^8C_4) + (1 \times {}^8C_3)$

$= 2016 + 630 + 56 = 2702$

(b) ज्ञात है कि,

the men are in majority in (i) only.

इसलिए,

No. of committees in which the men are in majority

$= \left({}^{9}C_{5} \times {}^{8}C_{7} \right)$

$= {}^{9}C_{4} \times {}^{8}C_{1}$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times 8$

$= 1008$.

(P.T.O.)

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Q. In a village, there are 87 families of which 52 families have at most 2 children. In a rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?

Sol. यह दिया है कि 52 families के पास ज्यादा से ज्यादा 2 children है।
यानी (87-52) = 35 families के पास 2 से ज्यादा children है।

हम 20 families select करनी है जिसमें कि कम से कम 18 families के पास ज्यादा से ज्यादा 2 children है।
selection ऐसे होगा :—

(i) 18 families out of 52 and 2 families out of 35
or (ii) 19 families out of 52 and 1 family out of 35
or (iii) 20 families out of 52.

80)

the number of ways in which these choices can be made.

$= ({}^{52}C_{18} \times {}^{35}C_{2}) + ({}^{52}C_{19} \times {}^{35}C_{1}) + ({}^{52}C_{20})$.

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Q. Find the number of ways in which a committee of 2 teachers and 3 students can be formed out of 10 teachers and 20 students. In how many of these committees

(i) a particular teacher is included?
(ii) a particular student is included?
(iii) a particular student is excluded?

Ans: No. of ways of forming a committee of 2 teachers and 3 students

$= \left({}^{10}C_2 \times {}^{20}C_3 \right)$

(i) No. of committes in which a particular teacher is included

अब क्योंकी एक टीचर पहले से included है तो बचा 9 टीचर उसमे से select करना है 1 क्योंकी 1 तो already selected है हुए है।

(P.T.O.)

∴ i) No. of committees in which a particular teacher is included

$= {}^{9}C_1 \times {}^{20}C_3$.

(ii.) No. of committees in which a particular student is included

अब क्योंकि एक student always
include करना है इसलिए बचे
19 students है। मे हमे दो बच्चे
select करना है

∴ No. of committees in which a particular student is included

$= {}^{10}C_2 \times {}^{19}C_2$

(P. T. O.)

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(iii) a particular student is excluded

तो इस case मे हमें एक student को हमेसा नही लेना है इसलिए,

$= {}^{10}C_2 \times {}^{19}C_3$

Q. How many different products can be obtained by multiplying two or more of the numbers 3, 5, 7, 11 without repetition?

Ans: देखो भाई product के लिए at least two numbers होने चाहिए यह बात हमे बहुत अच्छे से है मालूम

अब यहाँ हमारे पास 4 digits है इसलिए product इस प्रकार होंगे :-

taking 2 no.s	taking 3 no.s	taking 4 no.s
तो चार no. मे से 2 को select करना	तो तीन no. मे से 3 को select करना	तो चार no. मे से चार को select करना
${}^4C_2$	${}^4C_3$	${}^4C_4$

∴ Total no. of different products can be obtained by multiplying two or more of the members 3, 5, 7, 11

$= {}^4C_2 + {}^4C_3 + {}^4C_4$

$= 6 + 4 + 1$

$= 11.$

Q. How many different selections of 4 books can be made from 10 different books, if

(i) there is no restriction?

(ii) two particular books are always selected?

(iii) two particular books are never selected?

(P.T.O.)

(i) जब कोई restriction नहीं है तब मुझे 4 book 10 में से choose करना है

$$= {}^{10}C_4 = \frac{10!}{6! \cdot 4!}$$

(ii) जब दो book हमेशा choose करना है तो अब समझो की 2 book always selected है इसलिए अब हमको सिर्फ 2 book select करना है बचे हुए 8 books में से

$$= {}^{8}C_2$$

(iii) जब दो books हमेशा select नहीं करे तो अब बचे 8 books में से ही 4 book को choose करना है

$$= {}^{8}C_4$$

Q. A sports team of 11 students is to be constituted, choosing at least 5 from class XI and at least 5 from class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

Ans:

Class XI	Class XII
20 students	20 students

अब हमको sports team बनाना है जो 11 students से बना हो यह कुछ ऐसे कि 5 कम से कम बच्चा दोनों class का होना चाहिए तो दो तरीको से होगा

Case I: जब class XI से 6 बच्चे को select करे, और XII से 5 बच्चे को select करे।

$^{20}C_6 \times {}^{20}C_5$.

Case II जब class XI से 5 बच्चे को select करे और XII से 6 बच्चे को select करे।

Hence Total no. of ways in which team be constituted

$= {}^{20}C_5 \times {}^{20}C_6 + {}^{20}C_6 \times {}^{20}C_5$

$= 2 \cdot {}^{20}C_5 \times {}^{20}C_6$

Mixed Problems on Permutations and Combinations

Q. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

Ans: देखो, हमको question मे यह बोला है कि 7 consonants और 4 vowels मे से हम कितने words बना सकते है जिसमे तीन consonant हो और 2 vowel हो।

तो अब समझना कि हम को word बनाने के लिए दो काम करना पड़ेगा :—

(i) 7 consonants मे से तीन consonants को select करना (&) 4 vowels मे से 2 consonants को select करना

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(ii) Select किए 5 letters को आपस मे arrange करना

तो,

Total no. of ways of choosing (3 consonants out of 7) and (2 vowels out of 4)

$$= \left({}^7C_3 \times {}^4C_2\right) = \frac{7\times 6\times 5}{3\times 2\times 1} \times \frac{4\times 3}{2\times 1}$$

$= 210.$

~~= 210~~

No. of ~~words~~ groups each containing 3 consonants and 2 vowels
= 210.

∵ each group contains 5 letters.

∴ No. of ways of arranging 5 letters amongst themselves.

$= 5! = 5\times 4\times 3\times 2\times 1$
$= 120$

Hence,

The required no. of words

$= 210 \times 120$

$= 25200$ words

Q. The english alphabet has 21 consonants and 5 vowels. How many words with two different consonants and two different vowels can be formed from the alphabet?

Ans: question बोलता है कि तुम मुझे दिए गए 5 vowels और 21 consonants से कितने words बना के दे सकते हो कि उसमें 2 अलग-अलग consonants हो और 2 अलग-अलग vowels हो

तो final answer के लिए 2 काम करना पड़ेगा :—

(P.T.O.)

(i) selection of two different consonants out of 21 consonants (and) selection of two different vowels out of 5 vowels.

(ii) arranging the 4 selected letters amongst themselves.

Total no. of ways of choosing (2 consonants out of 21) and (2 vowels out of 5).

$= {}^{21}C_2 \times {}^{5}C_2 = 2100$

No. of groups, each containing 2 consonants and 2 vowels

$= 2100$

each group contains 4 letters.

No. of ways of arranging 4 letters amongst themselves

$= 4! = 4 \times 3 \times 2 \times 1 = 24$

Hence,

Required no. of words

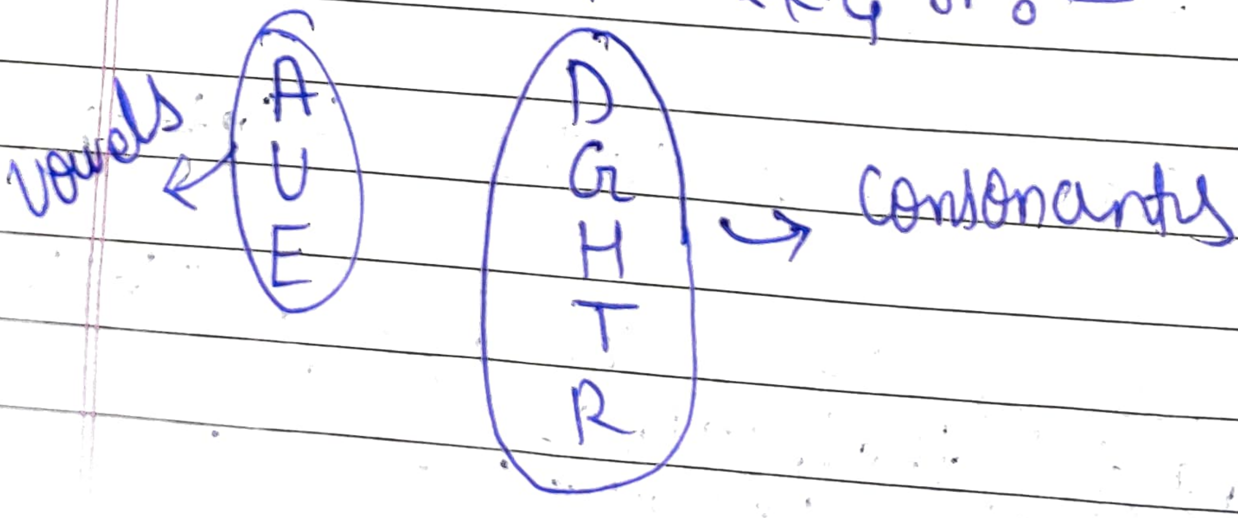
$= 2100 \times 24$

$= 50400$

Q. How many words with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word 'DAUGHTER'?

Ans: इस question मे बोला है कि तुम 'DAUGHTER' word मे से कितना words with / without meaning बना सकते है जिसमे 2 vowels हो और 3 consonants हो।

DAUGHTER शब्द मे :-



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अब देखो, total no. of words निकालने के लिए हमको दो काम करना है :–

(i) selection of 2 vowels out of 3 available vowels and selection of 3 consonants out of 5 available vowels.

(ii) arranging the 5 selected letters amongst themselves.

Total number of ways of choosing (2 vowels out of 3) and (3 consonants out of 5)

$$= \left({}^3C_2 \times {}^5C_3\right) = {}^3C_1 \times {}^5C_2$$

$$= 3 \times \frac{5 \times 4}{2 \times 1} = 30.$$

No. of groups, each containing 2 vowels and 3 consonants

$= 30$

Each group contains 5 letters.

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No. of ways of arranging 5 letters amongst themselves

$= 5! = (5 \times 4 \times 3 \times 2 \times 1)$
$= 120.$

Hence,

Required no. of words
$= 30 \times 120$
$= 3600$

Q. How many words with or without meaning, can be formed using all the letters of the word 'EQUATION' at a time so that vowels and consonants occur together?

(P.T.O.)

Ans:

5 vowels ← (E, U, A, I, O)   (Q, O, N) → 3 consonants

Total no. of words बताने के लिए हम को दो काम करना पड़ेगा :-

(i) ~~selection of~~ arrangement of 5 vowels amongst themselves (and) arrangement of 3 consonants amongst themselves.

(ii) Considering all vowels as 1 letter and all consonants as 1 letter, arranging these two packets.

दिए गए word 'EQUATION' में 5 vowels और 3 consonants है।

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सभी vowels एक साथ रखे जाएगे = 5! ways

सभी consonants एक साथ रखे जाएगे = 3! ways

मान लिया सभी vowels को वह एक letter है और सभी consonants को को वह एक letter है इसलिए vowels and consonants can be arranged in 2! ways

∴ required no. of words

$= (5!) \times (3!) \times (2!)$

$= 1440$

Q In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

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Ans: क्योंकी कोई भी दो boys साथ मे नही आने चाहिए इसलिए 5 girls को नीचे दिखाए way मे arrange कर दे :-

X G X G X G X G X G X

G → denotes the position of a girl
X → denotes the position of a boy

ये girls 5 places मे 5! ways मे arrange करो

अब; 3 boys को 6 places मे बैठाएंगे, each marked X

No. of ways of seating 3 boys at 6 places

$= {}^6P_3$

∴ required no. of ways

$= (5!) \times {}^6P_3 = 14400$

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Q. Out of 12 consonants and 5 vowels, how many words, each containing 3 consonants and 2 vowels, can be formed ?

Ans: हमे words बनाना है जिसमें 3 consonants और 2 vowels हो जिसको 12 consonants और 5 vowels से बनाना है

तो, total no. of words बनाने के लिए दो काम करना परेगा :-

(i) 3 consonants out of 12 consonants को select करना (and) 2 vowels out of 5 vowels को sect करना

(ii) चुने हुए 8 letters को आपस मे arrange करना

No. of ways of selecting 3 consonants out of given 12 consonants.

$= {}^{12}C_3$

No. of ways of selecting 5 vowels out of given 5 consonants

$= {}^{5}C_2$

No. of ways of arranging 5 letters amongst themselves.

$= (5 \times 4 \times 3 \times 2 \times 1) = 5!$

∴ Total no. of words containing 3 consonants and 2 vowels

$= {}^{12}C_3 \times {}^{5}C_2 \times 5!$

$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} \times \frac{5 \times 4}{2 \times 1} \times (5 \times 4 \times 3 \times 2 \times 1)$

$= 264,000$

Q. How many words, each of 3 vowels and 2 consonants, can be formed from the letters of the word 'INVOLUTE'?

Ans: letter 'INVOLUTE' में

4 vowels { I, O, U, E }

{ N, V, L, T } 4 Consonants

देखो अब question में हमको निकालना है कि कितना words जिसमें 2 consonants हो और 3 vowels हो बन सकता है। word 'INVOLUTE' से।

तो total no. of words बनाने के लिए दो काम करना पड़ेगा :-

(i) 3 vowels out of 4 vowels को select करना (and) 2 consonants out of 4 consonants को select करना

(ii.) 5 selected letters को खुद मे arrange करना।

$= 5! = 5 \times 4 \times 3 \times 2 \times 1.$

No. of ways of selecting 3 vowels out of 4 vowels

$= {}^4C_3.$

No. of ways of selecting 2 consonants out of 4 consonants

$= {}^4C_2.$

∴ Required no. of words

$= {}^4C_3 \times {}^4C_2 \times 5!$

$= \frac{4 \times 3 \times 2}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1} \times 5 \times 4 \times 3 \times 2 \times 1$

$= 64 \times 45$

$= 2880$ words

Q. The english alphabet has 21 consonants and 5 vowels. How many words with two different consonants and three different vowels can be formed from the alphabet?

Ans:

Total no. of words निकालने के लिए हम को दो काम करना पड़ेगा।

(i) 5 vowels में से 3 vowels को select करना and 21 consonants में से 2 consonants को select करना

(ii) चुने हुए 5 letters को आपस में arrange करना।

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No. of ways of selecting 3 vowels out of 5 vowels

$= {}^{5}C_{3}$

No. of ways of selecting 2 consonants out of 21 consonants

$= {}^{21}C_{2}$

No. of ways of arranging 5 words amongst themselves

$= 5 \times 4 \times 3 \times 2 \times 1 = 5!$

Hence

Total no. of words (ways) ~~in~~ which can be formed using 2 different consonants and three different vowels

$= {}^{5}C_{3} \times {}^{21}C_{2} \times 5!$

$= \frac{5 \times \cancel{4}^{2} \times \cancel{3}}{\cancel{3} \times \cancel{2} \times 1} \times \frac{21 \times \cancel{20}^{10}}{\cancel{2} \times 1} \times (5 \times 4 \times 3 \times 2 \times 1)$

= 252000

Q. In how many ways can 4 girls and 3 boys be seated in a row so that no two boys are together?

Ans: क्योंकि कोई भी दो boys साथ में नहीं आने चाहिए इसलिए 4 girls को नीचे दिखाए गए way में arrange कर दें:-

X G X G X G X G X

G → denotes the position of a girl
X → denotes position of a boy.

ये 4 girls को 4 places में 4! ways में arrange करो।

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अब ; 3 boys को 5 places में बैठाएंगे, each marked X.

No. of ways of seating 3 boys at 5 places

$= {}^5P_3$

∴ required no. of ways

$= 4! \times {}^5P_3$

$= (4 \times 3 \times 2 \times 1) \times (5 \times 4 \times 3)$

$=$

Total Combinations

★ **Number of ways of selecting one or more objects from 'n' different objects**

↳ इसका मतलब मेरे पास n different object है, इसका मतलब एक भी select कर सकता हूँ or two भी select कर सकता हूँ or 3 ... or upto n objects.

for example i have 10 different objects.
In how many ways i can select one or more out of these ten objects.
"one or more" का मतलब है one obj. select कर सकता हूँ or ~~three~~ two or three or four or five or six or seven or eight or nine or ten.

10 different objects

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 +$
${}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}$

$= \sum_{r=1}^{n} {}^{n}C_r$

or it can also be written

$= (2^n - 1)$

अब बड़ा सवाल भैया आपमें ऊपर वाला (2^n-1) होगा यह कैसे लाया तो देखो एक example :-

e.g.: there are 4 different objects

Total comb.s $= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

$= 4 + 6 + 4 + 1$

$= 15$

अब देखो 4 different object का total combination आया = '15'

अब इसी को अगर हम ऐसे लिखें तो :-

$= 2^4 - 1$ {तो 15 ही आएगा}

अभी हमने ऊपर $(2^n - 1)$ formulae की validity देखी की Manually निकाला हुआ answer और इस formulae को use कर के जो answer आया वह equal हुआ या नहीं

अब logic इस formulae का समझने के लिए हम इसे tree diagram में मिलेंगे

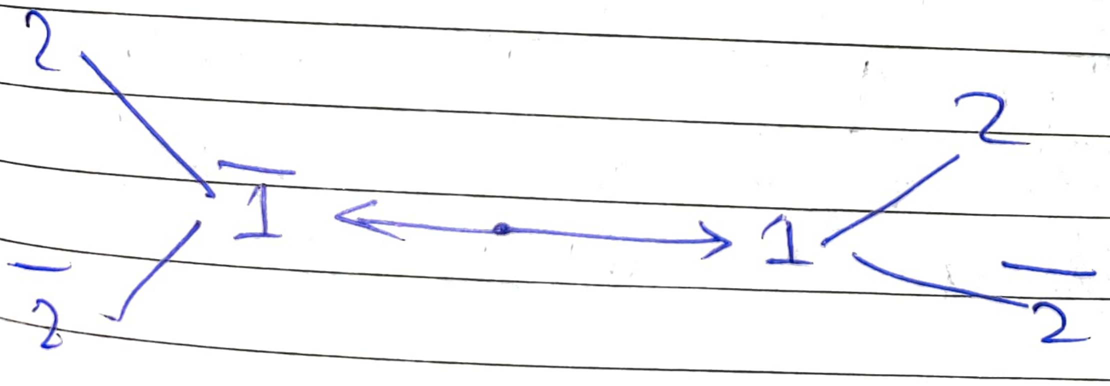
इस corresponding situation का tree diagram में draw कर रहा है।

तो मान लिजिए i have 4 objects; i can select 1 object (or) 2 objects (or) 3 objects (or) 4 objects

तो पहला object के लिए first object मैं ले भी सकता हूं या छोड़ भी सकता हूं, if you take a selection तो first object हो भी सकता है या नहीं भी हो सकता

1 नहीं भी हो सकता है { 1 ——— 1 } 1 हो भी सकता है

अब second object भी हो सकता है या नहीं भी हो सकता है इसलिए 1 के साथ 2 हो भी सकता है या नहीं भी हो सकता और अगर 1 नहीं है तो भी 2 हो सकता है या नहीं हो सकता है



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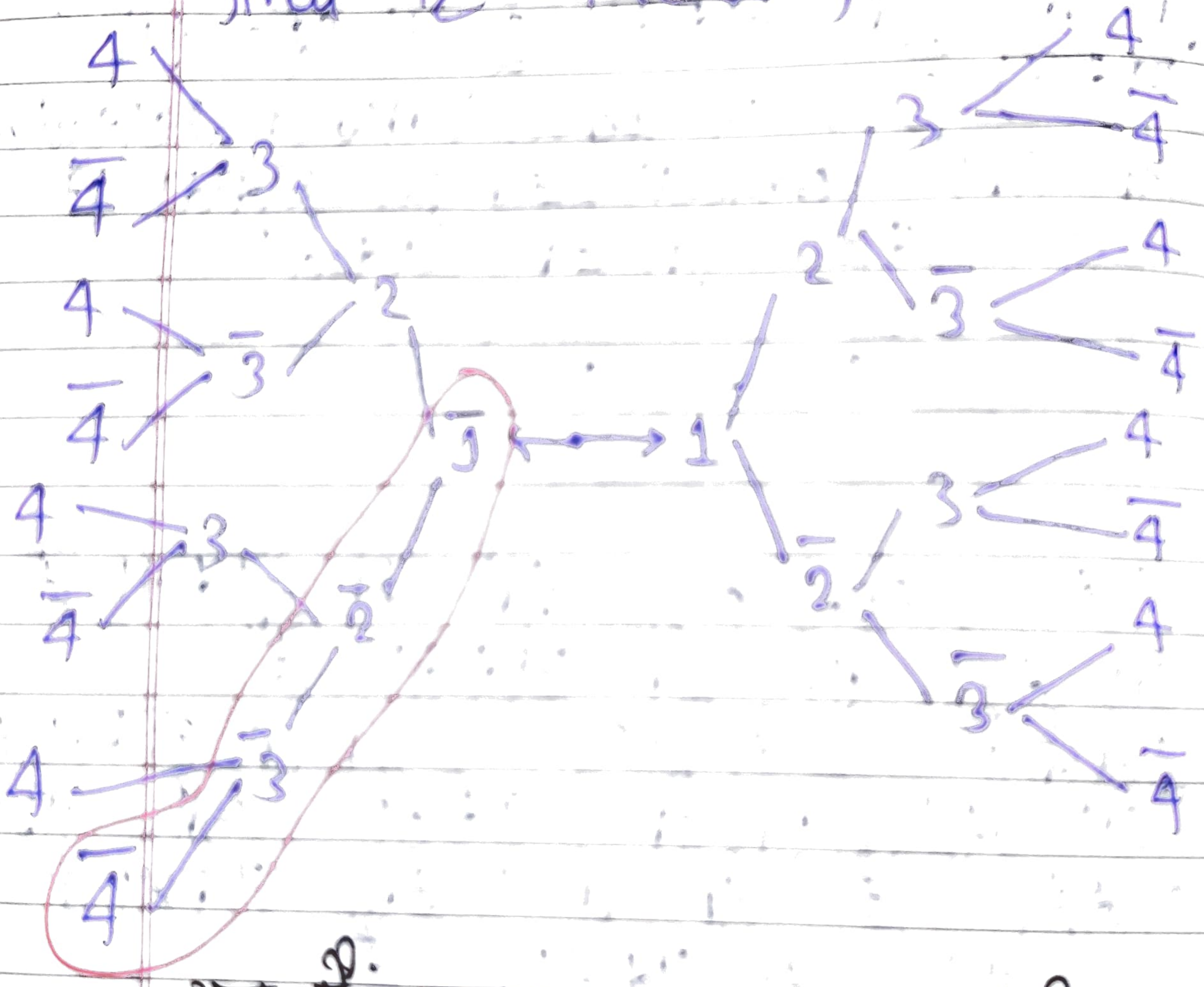
वैसे ही तीसरा और चौथा Object मे काम करते हुए हम final यह tree diagram पाएँ:-



अब मैं इस tree diagram की मदद से वह सारे selection लिख देता हूँ जो 4 distinct objects के possible होंगे :-

1. 1 2 3 4 → चारो selected
2. 1 2 3 $\bar{4}$ → 1, 2, & 3 selected
3. 1 2 $\bar{3}$ 4 → 1, 2, & 4 selected
4. 1 2 $\bar{3}$ $\bar{4}$ → 1&2, selected
5. 1 $\bar{2}$ 3 4 → 1, 3 & 4 selected
6. 1 $\bar{2}$ 3 $\bar{4}$ → 1& 3 selected
7. 1 $\bar{2}$ $\bar{3}$ 4 → 1&4 selected
8. 1 $\bar{2}$ $\bar{3}$ $\bar{4}$ → 1 selected
9. $\bar{1}$ 2 3 4 → 2, 3 & 4 selected
10. $\bar{1}$ 2 3 $\bar{4}$ → 2& 3 selected
11. $\bar{1}$ 2 $\bar{3}$ 4 → 2& 4 selected
12. $\bar{1}$ 2 $\bar{3}$ $\bar{4}$ → 2 selected
13. $\bar{1}$ $\bar{2}$ 3 4 → 3& 4 selected
14. $\bar{1}$ $\bar{2}$ 3 $\bar{4}$ → 3 selected
15. $\bar{1}$ $\bar{2}$ $\bar{3}$ 4 → 4 selected
16. $\bar{1}$ $\bar{2}$ $\bar{3}$ $\bar{4}$ → no one selected

Out of 16 selections there is one selection i.e the 16th selection where no objects is getting selected, लेकिन हमने heading दिया है no. of ways of selecting one or more objects. मतलब atleast one तो लेना पड़ेगा 8Q

This one branch corresponds to no selection ईसलिए minus one किया

$$\left(2^n - 1\right)$$

Hence,

No. of ways of selecting one or more objects from 'n' diff. objs.

$$= {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots\cdots + {}^nC_n$$

$$= \sum_{r=1}^{n} {}^nC_r$$

$$= \left(2^n - 1\right)$$

Combinations when all objects are not different

For example:
you have 5 A's (A A A A A) ; in how many ways can you choose 1 A ?
तो answer है 1 way
मान लो we have 10 A's (A A A A A A A A A A) ; in how many ways can you choose 1 A out of 10 A's?
तो answer है 1 way.
So,
if you have 'n' identical objects then no. of ways of selecting 1 identical object out of 'n' identical object is '1.'

अब question बदल दो no. of ways in which two ways can be selected?
तो फिर answer है 1 way.
देखो अगर तुम्हें अभी भी समझ में नहीं आया तो ऐसे समझो मैं न

ऐसा बोला की shop मे जाके
आप दो coffee bite लाइए तो
तुम shop मे गए तो obviously
shop मे दो नही होगा पूरा
डब्बा मे होगा तो दो ले के
आए तुम, मैंने बोला यह
इनका wrapper का colour
printing सही से नही है इसलिए
यह मुझे पसंद नही आया।
इसलिए दूसरा दो लेकर
आइए।

अब तुम गए और फिर दो लेकर
आए, अब मुझे यह कैसे पता
चलेगा की आप असली
मे जाकर change करके आए
या उसी को वापस दिखा
रहे है is there any way
in which i can find out
नही है ना
मतलब सब same है b/w all
Choclates are identical.

Hence,

No. of ways
no of the
identical

अब देखो
... पास

अब question
many ways
move objects
out of n

तो देखो इस
5 होगा
कर सकते
और हर
के लिए एक

Hence,

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objects

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Hence,

No. of ways of selecting any no. of objects out of 'n' identical objects = 1

अब देखो

तुम्हारे पास 5A's है (AAAAA)

अब question है मेरा की in how many ways in which one or more objects can be selected out of n given objects

तो देखो इस बार 1 नहीं बल्कि 5 होगा answer क्योंकि 1 भी select कर सकते or 2 or 3 or 4 or 5. और हर object को select करने के लिए एक ही रास्ता है

Hence,

~~No. of ways of selecting~~ in which one or more objects ~~can~~ be ~~selection~~ selected.
= 'n'

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Hence,

No. of ways in which one or more objects can be selected
= n

अब question है same previous example में की no. of ways in which zero or more objects can be selected इसमें तब one or more वाले answer में 1 add कर देंगे क्योंकि इसमें no selection भी ok है वही जो tree diagram में 16th selection आया था।

Hence

No. of ways in which zero or more objects can be selected

= n+1

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* No. of ways of selecting ~~zero~~ one or more objects when there are various group of objects where each group having similar kind of objects

example के लिए मान लो कि तुम्हारे पास एक box है उसमें 5 Mango bite chocolate है, 4 cofee bite chocolate है, 6 alpenlibe chocolate है, 7 hajmola है तो अब question है in how many ways you can select one or more objects chocolates out of this box.

इसमें the thing that you need to understand is ये 5 Mango bites आपस में same है; 4 coffee bites are identical है; 6 alpenlibe are identical है & 7 hajmolas identical है

लेकिन but 5 M.B is different from 4 cofee bites; 4 cofee bites different from 6 alpenlibe and so on.....

So there are 22 chocolates in total but ~~the~~ problem is all the 22 chocolates are not different अगर मान लिए सारे 22 chocolates different हैं then no. of ways of selection of one or more objects is $(2^{22}-1)$ क्योंकि no. of total combinations of one or more objects $=(2^n-1)$ लेकिन सारे 22 objects different नहीं हैं

Now,

Total no. of ways of selecting one or more chocolates

=

इस question को क्या करते हैं directly analysis करने के place में हम इसको एक mathematical problem में convert किया; So we have converted into this in a array problem

I take an array of 4 numbers

जब मैं यह chocolate box class के अंदर ले के आया तो मेरे box के अंदर Mango bite section में 5 हैं; coffee bite section में 4 हैं; alpenlibe section में 6 हैं & Hajmola section में 7 हैं।

~~मान~~

MB	CB	AB	H
5	4	6	7

मान लिजिए i have taken some no. of chocolates; obviously the difference will be the number of chocolates so mostly क्या है you just need to calculate once i selected the chocolates after that इस array के अंदर क्या क्या no's हो सकता है।

MB	CB	AB	H
↓	↓	↓	↓
6	5	7	8

(6) × (5) × (7) × (8)

Mango bite section

एक M.B. select कर सकता
या 2 M.B. select कर सकता
या 3 M.B. select कर सकता
या 4 M.B. select कर सकता
या 5 M.B. select कर सकता
या 0 M.B. select कर सकता

(इनमें से कोई एक कर सकते हैं)

So 6 ways से मैं M.B. select कर सकता हूँ

Coffee bite section

Zero C.B. select करें or
One C.B. select करें or
three C.B. ——— or
four C.B. ——— or
five C.B. ——— or
~~six C.B. ———~~ or

So 5 ways में मैं coffee bite को select कर सकता हूँ

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Alpenlibe section

0 selection (or)
1 selection (or)
2 selectn (or)
3 selection (or)
4 selection (or)
5 selection (or)
6 selection

So, total 7 ways से alpenlibe select कर सकते हैं

Hajmola section

0 selection (or)
1 selection (or)
2 selection (or)
3 selection (or)
4 selection (or)
5 selection (or)
6 selection (or)
7 selection

So, total 8 ways से Hajmola select कर सकते हैं

So, total ways in which this array can be filled

$$= (6 \times 5 \times 7 \times 8) - 1$$

यह एक 1 इसलिए Minus किया क्योंकि हम one or more selection की बात कर रहे हैं और हर section के लिए हम no selection या zero selection को भी consider किए हैं इसलिए एक condn ऐसा होगा की array के चारो column में zero होगा जिसका मतलब है no chocolates selected from the box.

जब zero selection allowed है तब minus one करने का जरूरत नहीं

(P.T.O.)

अब इसका formulae क्या होगा?

If there are
m identical things of type-1
n identical things of type-2
p identical things of type-3
q identical things of type-4

So now what is the ways of selecting one or more objects out of these objects

तो answer होगा

$$=(m+1)(p+1)(q+1)(n+1)-1$$

क्योंकी पिछले example मे हमने यह साफ देखा कि 5 M.B को select करने के 6 यानी (5+1) ways; 4 C.B. को select करने के 5 यानी (4+1) ways; 6 A.B. को select करने के 7 यानी (6+1) ways & 7 Hajmola को select करने के 8 यानी (7+1) ways.

★ No. of ways of selecting one or more objects when there various objects each having only one quantity.

मुझे से कि मान लिया कि एक box में 1 Cadbury bar है, 1 Snickers bar है, 1 Kit-Kat है & 1 five Star है।

तो अब हमको find out करना total no. of ways of selecting one or more objects

Cadbury	Snickers	Kit-Kat	Star

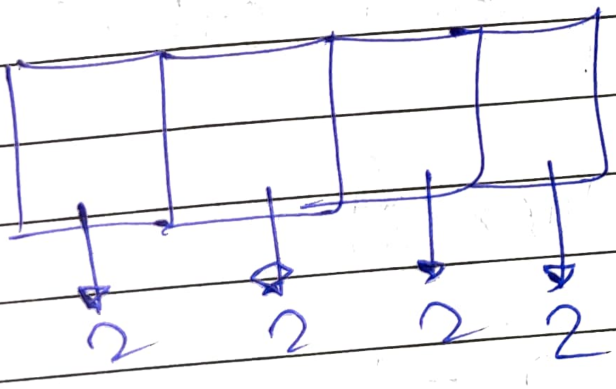
अब देखो पिछली बार की तरह हम इस प्रकार भी array बनाए और अब देखो

Cadbury को select करने के दो way
या तो select करेंगे या नहीं करेंगे।

Snickers को select करने के दो way
या तो select करेंगे या नहीं करेंगे।

Kit-Kat को select करने के दो way
या तो select करेंगे या नहीं करेंगे।

5-star को select करने के दो way
या तो select करेंगे या नहीं करेंगे।



$$= (2 \times 2 \times 2 \times 2) - 1$$

* अगर zero or more है तो 1 Minus
नहीं करेंगे।

अब इसका formulae क्या होगा?

If there are

total 'm' objects of diff type:
- a type — 1 obj
- b type — 1 obj
- c type — 1 obj
- m type — 1 obj

So now what is the no of ways of selecting one or more objects out of these objects

तो answer होगा

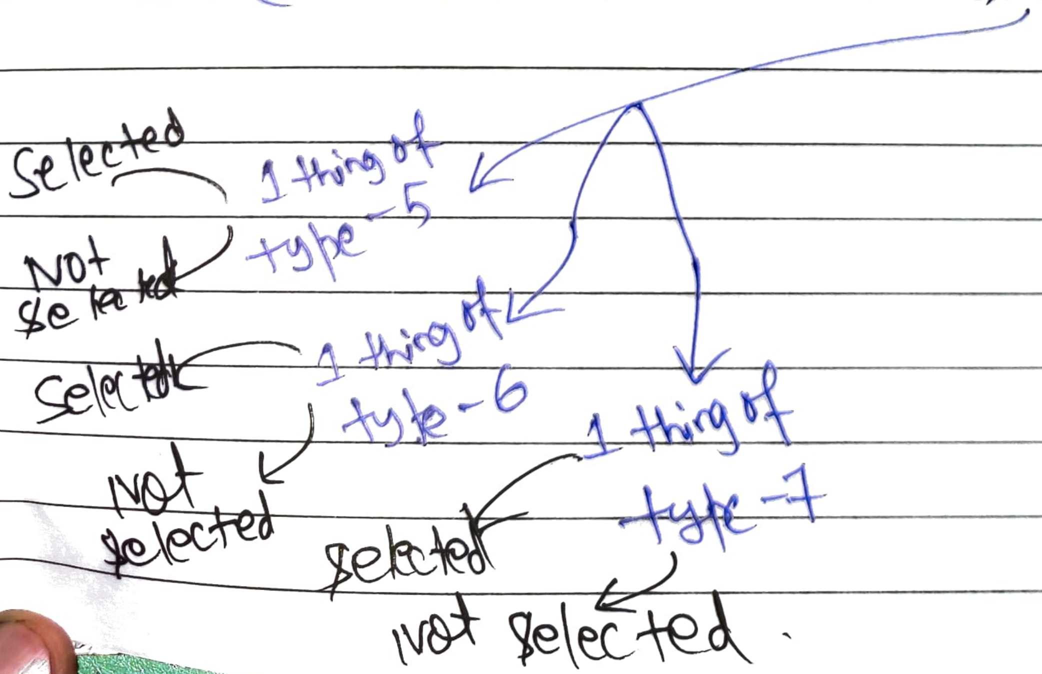
$$= 2^m - 1$$

where m is objects of different type

Now combinedly we can say

If there are

m identical things of type-1
n identical things of type-2
p identical things of type-3
q identical things of type-4
1 thing of type-5
1 thing of type-6
1 thing of type-7

$\therefore$ No. of ways of selecting one or more objects

$$= (m+1)(n+1)(p+1)(q+1)2^3 - 1$$